Section 2.5 Applications of Derivatives (Minimum homework: 1 – 9 odds)

- We will learn a few business applications that use Calculus in this section.
- Each application will involve a business that produces and or sells a single product.

We need to know the basic profit formula:

$$Profit = Revenue - Cost$$

We also need to understand what it means when we tack the word “Marginal” to the words Profit, Revenue and Cost.

Marginal Profit

- **Marginal profit** is the profit earned by a firm when one additional unit (or marginal unit) is produced and sold.
- The formula to compute marginal profit is the **derivative of a PROFIT formula.**
- Marginal profit is the difference between marginal revenue and marginal cost.
- $Marginal\ Profit = Marginal\ Revenue - Marginal\ Cost$
- Marginal profit analysis is helpful because it can help determine whether to expand or contract production or to stop production altogether, a moment known as the shutdown point.
- Under mainstream economic theory, a company will maximize its overall profits when marginal cost equals marginal revenue, or when marginal profit is exactly zero.

Marginal Cost

- Marginal cost is the additional cost associated with producing one more unit of a product.
- The formula to compute marginal profit is the **derivative of a COST formula.**

Marginal Revenue

- Marginal revenue is the increase in revenue that results from the sale of one additional unit of output.
- The formula to compute marginal revenue is the **derivative of a REVENUE formula.**

Marginal Profit Example:

A corporation determines the weekly profit (P(x)) from selling x units of a single product can be modeled by: $P(x) = -0.1x^2 + 20x - 36$

a) Find P(80)
b) Interpret your answer to part a. (round your answer to 2 decimals)
c) Create the marginal profit function P'(x) for this product.
d) Find P'(80).
e) Interpret your answer to part d.

a) $P(80) = -0.1(80)^2 + 20(80) - 36 = 924$
Answer: $P(80) = 924$

$b)$ The weekly profit will be $924 in a week in which 80 units of the product are sold.

c) $P'(x) = 2(-0.1)x + 20$
Answer: $P'(x) = -0.2x + 20$

d) $P'(80) = -0.2(80) + 20 = 4$
Answer: $P'(80) = 4$

e) An additional profit of $4 will be earned by producing and selling the 81st unit of the product.

Marginal Cost Example:

The cost function for producing x units of a certain product is:
$C(x) = -0.04x^2 + 80x + 2350$

a) Find C(10)
b) Interpret your answer to part a.
c) Create the marginal cost function C'(x) for this product.
d) Find C'(10)
e) Interpret your answer to question part d.

a) $C(10) = -0.04(10)^2 + 80(10) + 2350 = 3146$
Answer: $C(10) = 3146$

b) The total cost to produce 10 units of the product is $3,146.

c) $C'(x) = 2(-0.04)x + 80$
Answer: $C'(x) = -0.08x + 80$

d) $C'(10) = -0.08(10) + 80$
Answer: $C'(10) = 79.2$

e) It will cost about $79.20 to produce the 11^{th} unit of the product.

Price-demand function "explained"

- Demand depends on the price of a product.
- The higher the price, the less the demand. (generally)
- The lower the price, the more the demand. (generally)
- A price demand function describes a relationship between the demand (x) for a product and the price $p(x)$ for the same product.

Here is a simple price-demand function where

- x represents the number of units of a single product that are sold.
- $p(x) =$ price necessary to sell $x - units$ of that single product.

$p(x) = -1.25x + 50$

a) Let us evaluate the function at $x = 20$ and then explain the number generated.

$p(20) = -1.25(20) + 50$

$p(20) = 25$

$x = 20$ represents the number of units of the product sold.

$p(20) = 25$ represents the price of the product that will yield 20 units sold.

The Algebra basically tells us at a price of $25, 20 units of the product will be sold.

b) What is total revenue what will be generated when 20 units are sold at $25?

$$revenue = price * quantity$$

$$revenue = 25 * 20 = \$500$$

$500 of revenue will be generated when 20 units are sold for $25 each.

c) Use the price demand function to create a revenue function $R(x)$.

$$revenue = price * quantity$$

We $will\ use\ p(x) = -1.25x + 50\ to\ represent\ price$

We will use x to represent quantity.

$$R(x) = (-1.25x + 50)(x)$$

$$R(x) = -1.25x^2 + 50x$$

d) Use the revenue function to calculate the revenue that will be earned when 20 units are sold.

$$R(20) = -1.25(20)^2 + 50(20) = 500$$

$500 of revenue will be earned when 20 units are sold

Notice this is the same answer as question b. This is because this is another of way of asking the same question.

Marginal Revenue Example:

The price-demand function $p(x)$ for x-units certain product is given by the formula:

$p(x) = -1.50x + 75$
x is the number units of the product that are demanded
p(x) represents the price of the product in dollars.

a) Find p(10) round to 1 decimal.
b) Interpret you answer to part a.
c) Create a revenue function R(x) hint $R(x) = x * p(x)$ (revenue = quantity*price)
d) Find R(10).
e) Interpret your answer to part d.
f) Find the marginal revenue function R'(x).
g) Find R'(10).
h) Interpret your answer to part g.

a) $p(10) = -1.50(10) + 75 = 60$
Answer: $p(10) = 60$

b) 10 units of the product will be sold when the price is $60.

c) $R(x) = (-1.50x + 75)x$
Answer: $R(x) = -1.50x^2 + 75x$

d) $R(10) = -1.50(10)^2 + 75(10) = 600$
Answer: $R(10) = 600$

e) $600 of revenue will be earned when 10 units of the product are sold.

f) $R'(x) = 2(-1.50)x + 75$
Answer: $R'(x) = -3x + 75$

g) $R'(10) = -3(10) + 75$
Answer: $R'(10) = 45$

h) An additional $45 of revenue will be earned when the 11th unit of the product is sold.

1) The cost function for producing x units of a certain product is: $C(x) = 0.1x^2 + 8x + 100$,

a) Find C(100)

b) Interpret your answer to part a.

c) Create the marginal cost function $C'(x)$ for this product.

d) Find C'(100)

e) Interpret your answer to question part d.

2) The cost function for producing x units of a certain product is: $C(x) = 0.4x^2 + 7x + 8$,

a) Find C(4)

b) Interpret your answer to part a.

c) Create the marginal cost function $C'(x)$ for this product.

d) Find C'(4)

e) Interpret your answer to question part d.

Answers:
2a) $C(4) = 42.4$
2b) The total cost to produce 4 units of the product is $42.40
2c) $C'(x) = 0.8x + 7$
2d) $C'(4) = 10.2$
2e) It will cost $10.20 to produce the 5th unit of the product.

3) Suppose that the cost in dollars to make x cell phone cases is given by: $C(x) = \ln(x) + 2x$

a) Find C(100) (round to 2 decimals)

b) Interpret your answer to part a.

c) Create the marginal cost function $C'(x)$ for this product.

d) Find C'(100) (round to 2 decimals)

e) Interpret your answer to question part d.

4) Suppose that the cost in dollars to make a x pairs of socks is given by: $C(x) = \ln(x) + 0.75x$

a) Find C(50) (round to 2 decimals)

b) Interpret your answer to part a.

c) Create the marginal cost function $C'(x)$ for this product.

d) Find C'(50) (round to 2 decimals)

e) Interpret your answer to question part d.

Answers: 4a) $C(50) = 41.41$

4b) The total cost to make 50 pairs of socks is $41.41

4c) $C'(x) = \frac{1}{x} + 0.75$

4d) $C'(50) = 0.77$

4e) It will cost $0.77 or 77 cents to produce the 51st pair of socks.

5) Bob's Bobble heads company determines the profit function for producing and selling a certain bobble head can be modeled by: $P(x) = -0.001x^2 + 8x - 1000 \;\; 0 \leq x \leq 7000$. Where x represents the number of bobble heads sold and P(x) represents the monthly profit in dollars.

a) Find P(1000)
b) Interpret your answer to part a. (round your answer to 2 decimals)
c) Create the marginal profit function $P'(x)$ for this product.
d) Find P'(1000).
e) Interpret your answer to part d.

6) The Radio Corporation determines the weekly profit (P(x)) from selling x radios can be modeled by: $P(x) = -0.01x^2 + 12x - 2000 \;\; 0 \le x \le 1000$.

a) Find P(500)

b) Interpret your answer to part a. (round your answer to 2 decimals)

c) Create the marginal profit function $P'(x)$ for this product.

d) Find P'(500).

e) Interpret your answer to part d.

6a) $P(500) = 1500$
6b) The monthly profit is $1,500 in a month in which 500 radios are sold.
6c) $P'(x) = -0.02x + 12$
6d) $P'(500) = 2$
6e) An additional $2 of profit will be earned by selling the 501st radio.

7) A self-employed person determines that the weekly profit from his current vending machine route can be modeled by:
$P(x) = 10x - \sqrt{x} \quad 0 \leq x \leq 200$; where x represents the number of vending machines stocked and P(x) represents the weekly profit.

a) Find P(64)
b) Interpret your answer to part a. (round your answer to 2 decimals)
c) Create the marginal profit function $P'(x)$ for this product.
d) Find P'(64). (round to 2 decimals)
e) Interpret your answer to part d.

8) A telemarketing company has determined that the daily profit (P(x)) from selling x subscriptions can be modeled by:

$$P(x) = 15x + \sqrt{x} \quad 0 \le x \le 100$$

a) Find P(16)

b) Interpret your answer to part a. (round your answer to 2 decimals)

c) Create the marginal profit function P'(x) for this product.

d) Find P'(16). (round to 2 decimals)

e) Interpret your answer to part d.

8a) $P(16) = 244$

8b) The profit will be $244 in a day in which 16 subscriptions are sold

8c) $P'(x) = 15 + \frac{1}{2\sqrt{x}}$

8d) $P'(16) = 15.12$

8e) An additional profit of $15.12 will be earned by selling the 17^{th} subscription

9) A Sun City couple has a small garden, and they grow blueberries. They have found the price-demand function is: $p(x) = -0.50x + 6.50$

Where x is the number of quarts of blueberries demanded and $p(x)$ represents the price per quart in dollars.

a) Find p(5) round to 1 decimal.
b) Interpret you answer to part a.
c) Create a revenue function R(x) hint $R(x) = x * p(x)$ (revenue = quantity*price)
d) Find R(5).
e) Interpret your answer to part d.
f) Find the marginal revenue function R'(x).
g) Find R'(5).
h) Interpret your answer to part g.

10) A Boy Scout troop builds pinewood derby cars. They have found the price-demand function is: $p(x) = -0.50x + 25$

Where x is the number of pinewood derby cars demanded and $p(x)$ represents the price of a car in dollars.

a) Find p(10) round to 1 decimal.

b) Interpret you answer to part a.

c) Create a revenue function R(x)
hint $R(x) = x * p(x)$ (revenue = quantity*price)

d) Find R(10).

e) Interpret your answer to part d.

f) Find the marginal revenue function R'(x).

g) Find R'(10).

h) Interpret your answer to part g.

10a) $p(10) = 20$

10b) $at\ a\ price\ of\ \$20\ per\ car,\ 10\ cars\ will\ be\ demanded$

10c) $R(x) = -0.50x^2 + 25x$

10d) $R(10) = 112.50$

10e) The revenue will be $200 when 10 cars are sold.

10f) $R'(x) = -x + 25$

10g) $R'(10) = 15$

10h) An additional $15 of revenue will be earned when the 11th car is sold.